Valuation of Structured Products*

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February 3, 2014

Abstract

The market for structured products has grown dramatically in the past decade. Their diversity and complexity has led to the development of many different valuation approaches, and which approach to use to value a given product is not always clear. In this paper we demonstrate and discuss four approaches to valuing structured products: simulation of the linked financial instrument's future values, numerical integration, decomposition, and partial differential equation approaches. As an example, we use all four approaches to value a common type of structured product and discuss the virtues and pitfalls of each. These approaches have been practically applied to value 20,000 structured products in our database.

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Structured products have become a huge financial market. Bloomberg estimates that from 2010-2013 banks issued $174 billion in structured products in the United States and another $318 billion globally.[1] Structured products have evolved into highly complex instruments that incorporate features and conditions that are difficult to value. In addition, structured products are now often linked to not one but several underlying securities, making valuation all the more difficult.

While the academic literature has offered a variety of approaches for handling these features, no single valuation approach is appropriate for all structured products and the merits across these approaches are not always obvious.[2] In this paper we review the four primary approaches for valuing structured products, and offer our experience of valuing more than 20,000 products released over the past several years. We outline the advantages and disadvantages of each approach and identify the types of products most appropriate for each method. We stress that while similar valuations can often be obtained using several approaches, each approach offers different insight into the composition of the notes.[3]

We especially highlight how each approach handles the credit risk of the issuer. The 2008 bankruptcy of Lehman Brothers, one of the largest issuers of structured products, offered a dramatic demonstration of the importance of incorporating credit risk into product valuations. While it is widely recognized in the academic literature that structured products are sold at a substantial premium,[4] what is less appreciated is the magnitude of credit risk in those valuations. Tnvestors may continue to purchase structured products despite the known price premium because that they do not fully appreciate the inherent credit risk and mispricing of these investments.[5] We describe how credit risk is incorporated into each type of structured product valuation method, allowing practitioners an informed choice of valuation approaches.

In the following section we describe several complexities of structured products that can affect the choice of valuation approach. Section 3 explains the four valuation approaches and how they can handle each of these product features. Section 4 walks through the implementation of each of these approaches to value a common type of structured product, a Morgan Stanley Buffered PLUS. Section 5 describes our conclusions.

[1] Bloomberg Structured Notes Brief, January 6, 2011, January 5, 2012, 2012 Review and 2013 Outlook (January 3, 2013), and 2013 Year End Review (January 9, 2014).

[2] See for example Doebeli and Vanini (2010), Bernard and Boyle (2008), Bergstresser (2008), Bethel and Ferrel (2007), and Alexander and Venkatramanan (2009).

[3] The approaches described here can also be used to value structured certificates of deposit. The market for structured certificates of deposit is estimated to be in the tens of billions of dollars and growing, see Deng et al. (2013) for details. Variable annuity issuers are also releasing products based on structured product-like payoffs, which can be valued using similar approaches, as we describe in Deng et al. (2014a).

[4] See especially Deng et al. (2012) and Henderson and Pearson (2010).

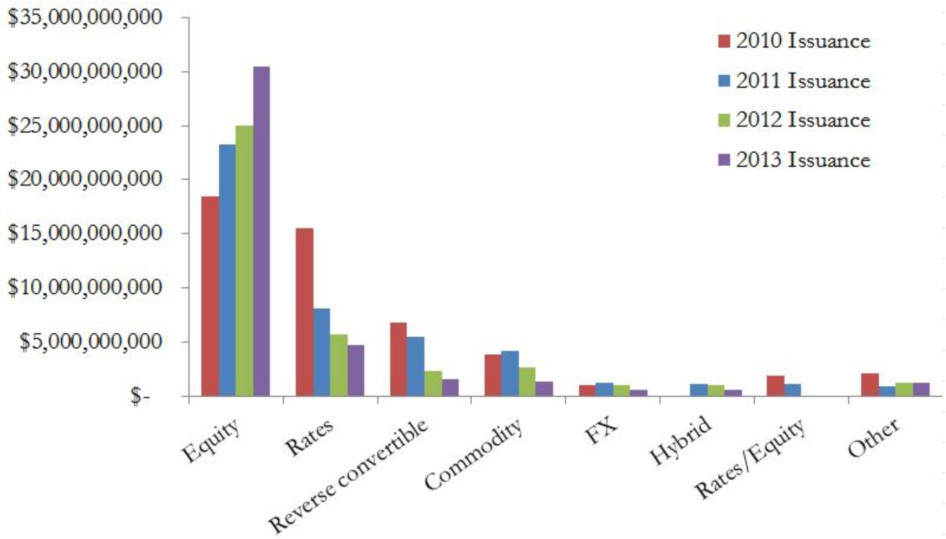
[5] See for example Deng et al. (2011b), Deng et al. (2011a), Deng et al. (2010), and Deng et al. (2009).

Common Structured Product Features

Structured products have varied and complex payoff structures. In this section we describe some of the characteristics that differentiate products.

UNDERLYING ASSETS Structured products are very often tied to the S&P 500, NASDAQ 100, or other broad stock index, but can also be linked to a very wide variety of other asset classes. Figure 1 shows the distribution of structured products released from 2010-2012 amongst various asset classes.[6]

Figure 1: Structured Product Issuances by Underlying Asset Class



It is important for investors to thoroughly understand the underlying asset or index to which a structured product is linked. Indexes are now available that use extremely complex and sophisticated investment strategies that can lead to unexpected risk and return profiles. For example, a 2008 structured product from Lehman Brothers was linked to an index that tracked a complex bundle of commodities that dynamically allocated between different assets based on conditions in their respective futures markets.[7] While this product had a simple payoff structure, the underlying index involved complex calculations of the returns to many assets, making this type of product particularly risky and difficult to model.[8] We have noted that more and more structured

[6] Source: Bloomberg Structured Notes Briefs dated January 5, 2012 (page 9), January 6, 2011 (page 11), and 2012 Review and 2013 Outlook (January 3, 2013, page 7). Reverse convertibles can be linked to a variety of underlying assets, but are listed separately because their payout can be in the form of the asset itself.

[7] Lehman Brothers Medium-Term Notes, Series I, 100% Principal Protected Notes Linked to ComBATS I due August 7, 2012. CUSIP: 5252M0GJ0

[8] For a more recent example, consider the $235 million JP Morgan Return Notes Linked to the J.P. Morgan

securities, including structured products and structured certificates of deposit, have been linked to proprietary indexes developed and calculated by issuers.

Valuation is easier when the underlying asset of a product has both an observable daily market price as well as an actively traded options market, as such assets have readily accessible correlation and implied volatility estimates. Other assets may require complex forward volatility and correlation models that can be difficult to implement and calibrate; in this paper, we assume that the underlying assets have reliable implied volatility and correlation information.

Structured products, like common derivatives such as options, generally link to the *price* of underlying assets, but do not give investors any dividends paid by those assets. For notes linked to assets paying high dividend yields, the payoffs are generally lower than they would be if the structured product were linked to the total return of the underlying asset rather than its price return.

PRINCIPAL PROTECTION

Principal protection absorbs a portion of the linked security's capital losses. Principal protection always requires that the structured product be held to maturity, and is void if the issuer defaults on the product. If a product is 100% principal protected, the investor will not lose money on the investment as long as the product is held to maturity and the issuer does not default. A product with more than 0% but less than 100% principal protection is called a "partial principal protected" or "buffered" product.

Principal protection and buffers are easily modeled with any of the approaches discussed here. Perhaps the most intuitive method is decomposition, where full principal protection on a long position could be modeled as an at the money put option. A buffer feature would then correspond to an additional short put option, where the degree of buffering could be thought of as the out-of-the-moneyness of the option. Our subsequent discussion of the Buffered PLUS product below includes treatment of buffers using all four approaches.

CONTINGENT CLAIMS

Some structured products have triggers, or specific return levels which yield contingent outcomes. For example, Citigroup's ELKS are simple bonds unless the trigger is tripped (the underlying security's returns surpass a predefined value), at which point the ELKS becomes a forward contract on the linked security. Other products, like absolute return barrier notes ('ARBNs', issued by UBS, HSBC, Deutsche Bank, and Lehman Brothers) have payoffs that depend on the size–but not the *direction*–of the linked security's return.[9] As long as the linked security's price does not go up or down by more than a pre-specified amount, the structured product pays the absolute value of the return. If the linked security's capital gain or loss exceeds the trigger, the investor earns a

Enhanced Beta Select Backwardation Alternative Benchmark Total Return Index due November 27, 2018, issued on November 22, 2013 (CUSIP: 48126NTE7).

[9] See Deng et al. (2011a)

0% return regardless of the linked security's future price movement. Most contingent claims, such as the ones in Buffered PLUS products, are clearly defined at a particular return level and can therefore be incorporated into any valuation approach.

COUPONS

Relatively few non-interest rate linked structured products pay coupons. Of those that do, most also have contingent claims, such as reverse convertibles and autocallables. Products without coupons instead rely on the linked security's capital gains to provide investors with a positive return. Coupon payments can be incorporated into any valuation approach, though for highly complex coupon payout formulas the simulation approach can often handle this feature most explicitly.

CALL FEATURES

Some structured products can be called by the issuer. Callable products generally guarantee a specified rate of return. Some products are called at the discretion of the issuer, while others are autocallable. Autocallable products are automatically called if a given criteria is satisfied on a predefined call date. Products can have embedded American or Bermudan call options. American options can be exercised by the issuer any time before the structured product's maturity date. Bermudan options can be exercised on specific dates during the life of the product (e.g. quarterly). The final call date is always the product's maturity date.

Call features can be among the most difficult product features to value. Autocallable products are relatively easier, as the call feature is predictable, but discretionary call features necessitate simulation approaches. A rigorous implementation of a simulation-based valuation for callable interest-rate linked structured products can be found in Andersen and Piterbarg (2010).

LEVERAGE

Structured products can give investors leveraged (or deleveraged) exposure to the underlying security's returns. Typically, the leveraged returns are only for a certain subset of possible returns, such as leveraged upside for returns between 0-10%. The decomposition approach illustrates this most clearly, as the issuer can just purchase more of the call or put option that generates the relevant return (two long call options, for example, generate 200% participation).

BASKETS

Some products are linked to combinations ('baskets') of two or more linked securities. The payoff of the product is normally linked to the basket as a whole. However, some products allow the issuer to use the worst performing member of the basket to determine the product's payoff. When products are linked to baskets, any valuation must also take into account the co-movement among the basket members. There is considerable leeway in calculating this co-movement, but it is typically the correlation of daily returns of the underlying basket members over a reasonably large period of time, perhaps three months.[10] However the exact length of time is a subjective judgment, as the correlation structure between basket entities can change with time, but a suitable sample size must be achieved.

A structured product may have several or none of these characteristics. For example, Morgan Stanley's Performance Leveraged Upside Security (also called a PLUS) pays no coupon and is usually not callable. The PLUS exposes investors to all the downside risk of a single linked security along with levered but often capped upside potential. Some PLUS products offer partial principal protection. This partially principal-protected PLUS is sold as a Buffered PLUS by Morgan Stanley, a Partial Protection Return Optimization Security by UBS and Lehman Brothers, and an Equity Buffer Note by HSBC. For continuity, we will use a Buffered PLUS as the example throughout this paper.

Different product types have somewhat different return characteristics. In Deng et al. (2014b), we calculated an index of ex post structured product returns by valuing over 20,000 products each day from 2007 through 2013. We also calculated sub-indexes based on four common structured product types: autocallables, tracking securities, reverse convertibles, and single-observation reverse convertibles. Our results for both the aggregate and sub-indexes demonstrate that structured products as a whole have high correlations with equity markets, though lower returns. Those lower returns are in large part attributable to the issue date mispricing.

Valuation Approaches

In this paper, we discuss four structured product valuation approaches: 1) simulation, 2) numerical integration, 3) decomposition and 4) partial differential equations (PDE) approaches. While these approaches produce consistent valuations for simple products, not all approaches are appropriate for all products and they can vary in complexity and difficulty of implementation.

In general, structured product payoffs can be described as functions of the underlying security's level (e.g., stock price) or return. For example, the payoff rule of a Buffered PLUS is a function of the underlying security's level or holding period return at the structured product's maturity. In this paper, we use $P(S_T)$ as the functional form of payoff rules that are a function of the underlying security's ending stock price.

Because the underlying security's *holding period return*, R_t, is a function of the security's stock price

$$R_t = \frac{S_t - S_0}{S_0},$$

the functional form of the payoff rule can also be expressed in terms of the underlying security's return. We use $f(R_T)$ as the functional form of payoff rules that depend on the underlying security's return over the life of the structured product, where $P(S_T) = I \cdot (1 + f(R_T))$ and I is

[10] One approach to approximating the value of basket options is presented in Alexander and Venkatramanan (2009).

the face value of the structured product. The Buffered PLUS' payoff rule, graphed in Figure 2, can be expressed algebraically as

$$f(R_T) = \begin{cases} \min(\alpha R_T, Cap), & \text{if } R_T \geq 0; \\ \min(R_T + Buffer, 0), & \text{if } R_T < 0. \end{cases} \tag{1}$$

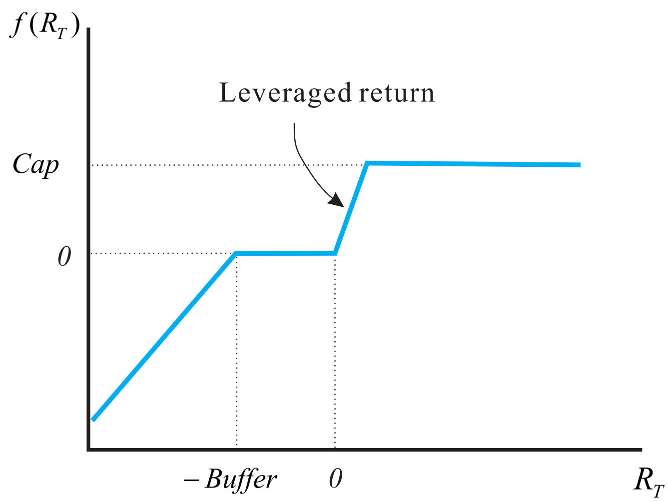


Figure 2: Payoff Rule for Buffered PLUS Structured Products

Assumptions Common to all Four Approaches

Each of the four approaches discussed in the following sections relies on a common set of assumptions. In this section we briefly discuss each assumption.

Generalized Wiener Process of Stock Prices

We assume stock prices follow a generalized Wiener process with a fixed drift (Glasserman, 2003; Hull, 2011; McLeish, 2004). The Wiener process is algebraically denoted as

$$dS_t = S_t \mu dt + S_t \sigma dZ_t, \tag{2}$$

or equivalently

$$d \ln S_t = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma dZ_t, \tag{3}$$

such that S_t is a geometric Brownian motion.

The generalized Wiener process results in an ending stock price S_T that is log-normally distributed

$$S_T \sim S_0 \cdot \text{Log-}\mathcal{N}\left(\left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right), \tag{4}$$

which is equivalent to

$$\ln(S_T) \sim \ln S_0 + \mathcal{N}\left(\left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right), \tag{5}$$

where $\mathcal{N}(\hat{\mu}, \hat{\sigma})$ is a normal distribution with mean $\hat{\mu}$ and standard deviation $\hat{\sigma}$. This directly implies that the holding period return $R_T = \frac{S_T - S_0}{S_0}$ must also be log-normally distributed

$$R_T \sim \text{Log-}\mathcal{N}\left(\left(\mu - \frac{\sigma^2}{2}\right)T, \sigma\sqrt{T}\right) - 1. \tag{6}$$

The Underlying Security's Expected Return

Like the Black-Scholes model, we value structured products in a risk-neutral framework (Bjork, 2004; Hull, 2011). Consequently, the expected annualized return for a structured product's underlying security is based on the risk-free rate r.[11] Because structured product payoff rules ignore dividends, we follow the dividend modification of the Black-Scholes model and reduce the risk-free rate by q, the dividend yield on the underlying security. This means that the underlying security's risk-neutral expected return, μ, is

$$\mu = r - q. \tag{7}$$

The Underlying Security's Implied Volatility

The volatility, σ, of the linked security's return is typically estimated as the volatility needed to make traded options fairly priced using the Black-Scholes model. Where possible, we match the option's and structured product's time to expiration. In cases where the structured product's remaining term falls between two option maturities, we use linear or quadratic interpolation to estimate the implied volatility. In the rare case when no options can be found for the underlying security, we use the underlying security's historical volatility.

Assumptions about the Discount Rate

To estimate the issue date value of a structured product, the expected cash flows must be discounted back to time $t = 0$. Because the structured product is exposed to both the risk of the underlying security and the issuer's credit risk, we include the issuer's credit default swap spread (CDS) in the discount rate (Hull, 2011). We match the term of the CDS with the term of the structured product.

[11] For simplicity, all returns in this paper are annualized and continuously compounded.

The Simulation Approach

Simulation has been proven to be a practical, useful tool for valuing financial products (Glasserman, 2003; McLeish, 2004). Of the four approaches discussed in this paper, the simulation approach is the most "brute force" in that it relies on computing power to perform Monte Carlo simulations of the linked security's price path. The Monte Carlo results are then input into the product's payoff rule to calculate the exact cash flows that would result from each simulated price path. The structured product's estimated fair value is the average of the discounted cash flows derived from the simulations. For structured product valuations, we use widely-accepted financial models to simulate security levels and returns, interest rates, and exchange rates.

To simulate stock prices following the generalized Wiener process, suppose we track stock prices at discrete time intervals $S_0, S_{t_1}, \ldots, S_{t_i}, S_{t_{i+1}}, \ldots, S_T$, and constant interval

$$\Delta_t = t_{i+1} - t_i.$$

At each step, we assume the stock price updates according to

$$S_{t_{i+1}} \quad = \quad S_{t_i} \cdot e^{\mu \Delta t + \sigma \sqrt{\Delta t} W_i} \tag{8}$$

where W_i is a standard normally distributed variable.

To value a structured product, we typically simulate $J = 50,000$ price paths of the underlying security from $t = 0$ to $t = T$. On the jth trajectory, suppose the ending stock price is S_T^j and the holding period return is R_T^j. The structured product's payoff for the jth simulated price path is calculated by inputting S_T^j into the mapping rule $P(S_T)$ or R_T^j into $f(R_T)$. We repeat the process for each of the 50,000 simulated price paths and calculate the average payoff. The average payoff is then discounted back to time $t = 0$ as shown in Equations (9) and (10).

$$PV(S_T) \quad = \quad e^{-(r+CDS)T} \frac{1}{J} \sum_{j=1}^{J} P(S_T^j) \tag{9}$$

$$PV(R_T) \quad = \quad e^{-(r+CDS)T} I \left(1 + \frac{1}{J} \sum_{j=1}^{J} f(R_T^j) \right). \tag{10}$$

The discounted expected payoff is the issue date value of the structured product.

Except for products with explicit American or Bermuda options, we have been able to value any structured product using the simulation approach.[12] In addition to being versatile, the simulation

[12] The American and Bermuda options mentioned here are explicitly part of the payoff rule. For example, some structured products allow the issuer to call the product at various dates prior to maturity.

approach is intuitive and easy to understand. The benefits of the simulation approach are offset, however, by the extensive programming time and computing power needed to perform a simulation, as well as the potential lower accuracy of the resulting fair values.

The simulation approach allows us to value structured products that are linked to a wide variety of securities including individual stocks, indices, interest rates, currency exchange rates or baskets of securities. The academic literature is constantly making advances in simulation efficiency, such as quasi-Monte Carlo and importance sampling approaches (Glasserman, 2003).

The simulation method can also be applied to calculate 'Greeks,' the sensitivity measures of derivative financial instruments. The Greeks are useful for traders to hedge risks in a portfolio containing structured products. The general procedure for calculating Greeks using simulations is to introduce an infinitesimal change in a chosen parameter, such as the risk free rate, volatility, etc, and to measure the resulting change in the fair value of the product. Since simulation introduces random noise in fair value estimations, fixing common random seeds is a must in running simulations. This practice can significantly improve the accuracy of estimating Greeks.

Besides using the generalized Wiener process to simulate stock prices,[13] we also use it to simulate currency exchange rates, where S_0 is the spot exchange rate, the "expected return," μ, is the difference between the risk-free rates of the countries whose currencies are included in the exchange-rate forward contract, and the volatility, σ, is the implied volatility of the exchange rate. The generalized Wiener process results in log-normally distributed currency exchange rates, like it does for stock and index returns. To simulate short interest rates we use the Cox, Ingersoll and Ross model in Cox et al. (1985). To simulate forward interest rates we use the Heath, Jarrow and Morton model in Heath et al. (1992). Of course, other interest models (such as the Libor Market Model) can be used as well.[14]

The Numerical Integration Approach

The numerical integration approach values structured products more quickly and more accurately than the simulation approach (Glasserman, 2003), but only works for a subset of structured products. Numerical integration directly utilizes the fact that the structured product's payoff rule is a function of a variable with a known distribution, such as S_T (see Equation (4)) or R_T (see Equation (6)).

The numerical integration approach is generally more accurate than the simulation approach because, unlike simulations which rely on the Law of Large Numbers to produce reliable valuations, the numerical integration approach considers the probability of virtually all possible outcomes by integrating the product of the return distribution and the payoff rule. As in the simulation approach, given the structured product's payoff rule and the distribution of the underlying security's

[13] When the linked security is a basket of securities, we use a multi-variate generalized Wiener process and include the entire variance-covariance matrix of the securities in the simulation.

[14] For an extensive treatment of interest rate models, see Andersen and Piterbarg (2010).

return, the present value of the structured product is

$$PV(R_T) = e^{-(r+CDS)T} I \left(1 + \int_{-1}^{\infty} f(R_T) pdf(R_T) dR_T\right). \tag{11}$$

Note that Equation (10) converges to Equation (11) as $J \to \infty$.

There are many valid numerical integration methods, such as the adaptive Simpson method using the Simpson quadrature (McKeeman, 1962) and the adaptive Lobatto method (Ueberhuber, 1997). Numerical integration approaches differ in how they generate sample points and assign weights to each sample, but should all estimate the same fair value. More advanced numerical integration approaches involve numerical expansions, such as Fourier series expansion (Carr and Madan, 1999) or Fourier cosine series expansion (Fang and Oosterlee, 2008). When the underlying return has a smooth distribution, such as a normal distribution, the expansion terms could achieve an exponential convergence rate to the integral value. High accuracy results are typically obtained with less than 100 expansion terms.

One drawback to the relatively fast and accurate numerical integration approach is that its use is limited to structured products whose payoffs are a function of variables with known distributions. This can be a problem, for instance, when valuing products whose payoffs depend on the linked security's maximum or minimum price over the life of the structured product. For structured products with these path-dependent payoff functions,[15] the numerical integration approach can be used, but requires transforming the path-dependent payoff function into an equivalent portfolio of path-independent payoff functions. Published papers such as Breeden and Litzenberger (1978) and Carr and Chou (1997) demonstrate how to transform the path-dependent payoff rules.

Using the numerical integration approach, the Greeks are calculated by taking derivatives of the payoff integral with respect to parameters such as S_0 (the initial price) or r (the risk-free rate).

For example, consider delta, the sensitivity of the structured product's value to the stock price S_t:

$$\Delta_t = \frac{\partial PV}{\partial S_t}. \tag{12}$$

A transformation will simplify the integral. We introduce a new variable $W = \frac{\ln S_T - \hat{\mu}}{\hat{\sigma}}$, where $\hat{\mu} = \ln S_t + (\mu - \frac{\sigma^2}{2})(T-t)$ and $\hat{\sigma} = \sigma\sqrt{T-t}$ are the parameters of the distribution of $S_T|S_t$. The variable W is thus a standard normal variable. It follows that

$$\Delta_t = \frac{\partial PV}{\partial S_t}$$

[15] A path-dependent payoff function depends on one or more historical prices over the life of the structured product and can be written as $P(\vec{S_t}), t = [0, t_1, t_2, \ldots, T]$. A path-independent payoff function depends only on the final price, S_T, of the underlying security.

$$
\begin{aligned}
&= \frac{\partial}{\partial S_t}\left(e^{-(r+CDS)T}\int_{-\infty}^{\infty} P(e^{W\hat{\sigma}+\hat{\mu}})pdf(W)dW\right) \\
&= e^{-(r+CDS)T}\int_{-\infty}^{\infty} P'(e^{W\hat{\sigma}+\hat{\mu}})\frac{e^{W\hat{\sigma}+\hat{\mu}}}{S_t}pdf(W)dW.
\end{aligned}
$$

If $P(S_T)$ is a piecewise linear function (as in Buffered PLUS), then $P'(S_t)$ is a Heaviside step function.

The Decomposition Approach

The decomposition approach can be applied to any product for which the payoff can be decomposed into a combination of conventional debt instruments, call and put options, and exotic options such as double-barrier options. After the payoff of a structured product is broken down into an equivalent portfolio of simpler financial instruments, each component of the portfolio is valued using the appropriate formula (such as the Black-Scholes model for valuing call and put options). This method is faster than the simulation method, uses less computing power, and does not require that the product's underlying security have a continuous, integrable return distribution like the numerical integration method. However, because not all structured product payoff rules can be broken down into components with simple formulaic solutions, the decomposition only works for a subset of structured products.

A significant advantage of the decomposition approach is its usefulness in characterizing a structured product's payoff rule in terms of other financial products. This can help investors understand the risks involved in the product, and see how they might create a more liquid version of the product by investing in the equivalent portfolio rather than the structured product.

In addition to plain-vanilla options, decompositions can also involve more complex, exotic options such as "down-and-in" single-barrier options (Hernández et al., 2007; Szymanowska et al., 2009). Down-and-in put options are inactive (do not pay out at maturity) unless a lower barrier is breached. If the underlying security ever goes below the lower barrier, the options become active puts. Allowing exotic options like the down-and-in put options in the structured product's equivalent portfolio expands the number of structured products that can be valued with the decomposition approach.

Like the numerical integration approach, the decomposition approach is useful for calculating the structured products 'Greeks'. For example, the delta of the structured product is simply the sum of the deltas of each component.

The PDE Approach

The PDE approach can be used to value any structured product, but is by far the most difficult to implement. Like the simulation approach, PDE captures the dynamic nature of the structured product's and linked security's values (Black and Scholes, 1973; Henderson and Pearson, 2010;

Wilmott et al., 1994). However, rather than generating random numbers to predict the movement of the linked security's price, the PDE approach models the relations of product values and stock price with partial differential equations. The various features of a structured product are included in PDE as additional equations, or boundary conditions. Since PDEs with several boundary conditions are relatively difficult to solve, only a handful of structured products result in closed-form solutions. The remaining product types rely on numerical methods such as finite difference approximation to obtain an approximate solution.

Like the simulation approach, the PDE approach assumes the underlying security price follows a generalized Wiener process (see Equation (2)). If the risk-free interest rate and the volatility are assumed to be constant across time, the PDE satisfies the Black-Scholes equation

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r-q)S\frac{\partial V}{\partial S} - (r + CDS)V = 0, \tag{13}$$

where the value of structured product $V(S,t)$ depends on the the stock price $S \in [0, \infty)$ at time $t \in [0, T]$. After solving the equation, the issue date value of the structured product is simply $V(S_0, 0)$.

The various properties of a structured product are included as boundary conditions in the Black-Scholes equation. A few examples:

- At maturity, the market value of the structured product is equal to the product's payoff. The corresponding boundary condition is

$$V(S,T) = P(S_T).$$

- If the security price hits 0 at time t and the structured product is principal protected, the structured product's value is the present value of its face value. The boundary condition is

$$V(0,t) = I \exp^{-(r+CDS)*(T-t)}.$$

  If the product is not principal protected, the structured product's value is the present value of the protected portion of its face value. The boundary condition is

$$V(0,t) = I\mathit{Buffer} \cdot \exp^{-(r+CDS)*(T-t)}.$$

- If the structured product can be called by the issuer, the structured product will never be worth more than the call price $\hat{C}_t$ on the call date t. The boundary condition is

$$V(S,t) \leq \hat{C}_t.$$

- If the linked security pays a D dividend on ex-dividend date t_d, the value of the structured product must be the same just before and after the dividend is paid. The condition is included as

$$V(S, t_d^-) = V(S - D, t_d^+),$$

where S remains constant from t_d^- through t_d^+, and t_d^- and t_d^+ are just before and just after the dividend is paid, respectively.

- If the structured product makes a coupon payment C at time t_c, the value of the structured note is reduced by C. At time t_c, the value is updated

$$V(S, t_c^-) = V(S, t_c^+) + C.$$

The solution methods for the partial differential equations falls into two categories: closed form solution and numerical solution. Unfortunately, majority of the partial differential equations can only be solved numerically.

PDE produces a closed-form solution when the system of equations, including the boundary conditions, is sufficiently simple and can be transformed into standard partial differential equations (e.g., parabolic equation, through a change of variables.[16]). We show how this is done when we value a Buffered PLUS in Section . Fourier transforms and other transforms are also helpful in solving the equation in closed-form, with the solution being expressed in infinite summation form of eigenfunctions (Hui, 1996).

Numerical methods are of various types, such as finite difference method and finite element methods. Finite difference method is the most popular choice. One involves approximating the partial differential terms $\frac{\partial V}{\partial t}, \frac{\partial^2 V}{\partial S^2}$, and $\frac{\partial V}{\partial S}$ with finite difference terms. The are three basic finite difference approximations - explicit, inexplicit and Crank-Nicolson method. The θ-method, uses an interpolating parameter θ to transform among these three methods.

Comparison of Valuation Approaches

While many products can be valued using any of the approaches described above, some structured product features make certain valuation approaches more difficult than others. Also, some valuation approaches provide a stronger intuition for the underlying features or offer simpler implementation. We outline in Table 1 our experience regarding which valuation approaches are most appropriate for several common product features including those we describe in Section . The choice of valuation approach depends on the level of accuracy required, implementation difficulty (especially the existence of closed-form solutions), and the overall purpose of the valuation itself.

The most important factor affecting the choice of valuation approaches is whether the payoff of the structured product is linked just to the final value of the underlying asset at maturity, or the specific *path* of values the asset takes over the life of the note. Path-dependent product features, such as call features, are much less analytically tractable and typically require a simulation based

[16] The process is called 'dimensionless' in Wilmott et al. (1994).

Table 1: Preferred Valuation approaches for Several Product Features

	Simulation	Numerical Integration	Decomposition	PDE
Callable (European) by Issuer	+	+		+
Callable (American) by Issuer				+
Callable (Bermudan) by Issuer	+			+
Maximum Allowable Return	+	+	+	+
Loss Buffer	+	+	+	+
Single-barrier (e.g. ELKS)	+		+	+
Double-barrier (e.g. ARBN)	+		+	+
Payoff Depends on Average Price	+	+		
Basket of Linked Securities	+	+	+	
Linked Security is a Currency	+		+	+
Pays Coupons	+	+	+	+
Minimum of a Basket	+			

approach. Structured products that only depend on final values, however, offer more flexibility. For example, principal protection can be implemented in any valuation method, but the decomposition approach most clearly demonstrates the combination of options used to generate the resulting payoff, and can therefore be most directly compared to actual options prices if such a market exists on the underlying.

Our discussion of these approaches so far has been based on a geometric Wiener process for asset prices (Equation (2)). However, recent developments in financial modeling extend the constant volatility assumption in this model to a stochastic volatility setting. Standard stochastic volatility models include the Heston model, where time varying volatility is introduced following a CIR (Cox-Ingersoll-Ross) type mean-reversion process (Heston, 1993). More recent models include general Lévy based price models (Schoutons, 2003) where each change in an asset's price is considered as an instantaneous jump. Stochastic volatility models can capture more detailed features of stock returns, such as skewness and kurtosis, than the traditional constant volatility model.

The simulation approach offers the most flexibility, and has been practically used in all varieties of stochastic volatility models to simulate asset prices (Korn et al., 2010). However, this approach can be computationally intensive. For example, when simulating the Heston model, two processes need to be simulated (the asset price process and the volatility process). Stochastic volatility has also been applied to the numerical integration approach in recent years. Since all asset return information is defined in a characteristic function, the typical integral form involving a return density function (Equation (11)) can be transformed into a series of summation terms using simple characteristic function valuations (Fang and Oosterlee, 2008). Therefore, switching between different return models involves simply plugging in corresponding characteristic functions. The decomposition approach retains its natural simplicity, since valuations of vanilla options as well as simple exotic options are readily available in closed form using stochastic volatility. While the first three approaches can easily be extended to incorporate stochastic volatility, the PDE approach is more restrictive. The only stochastic volatility model that can be expressed in the PDE approach is the Heston model (Heston, 1993).

An Example with All Four approaches

In this section we demonstrate how to use each of the four approaches to value a Buffered PLUS issued by Morgan Stanley on December 31, 2008 (CUSIP: 617483797). The Buffered PLUS has a two-year term ($T = 2$) and is based on the S&P 500 index. The return mapping function for this product is presented algebraically in Equation (1) and graphically in Figure 2. The product has a leverage ratio of 2 ($\alpha = 2$), a maximum allowed return of 60%, and a loss buffer of 10%. This means that investors in this product lose money if the S&P 500 index drops more than 10% during the two-year period, earn a 0% return if the S&P 500 index loses between 0% and 10%, and make money if the S&P 500 index increases.[17]

We collect the necessary variables from Bloomberg. On the pricing date, the S&P 500 index level was $S_0 = 863.16$ and the index's 24-month implied volatility was $\sigma = 37.75\%$. We assume the dividend yield will remain constant over the two years and use the annual dividend yield $q = 3.714\%$ for each year. The continuously compounded 2-year treasury spot rate is $r = 0.850\%$, and the 2-year CDS quote for Morgan Stanley is $CDS = 5.209\%$.

Using the Simulation Approach

To apply the simulation approach, we simulate monthly index levels $S_{t_0}, S_{t_1}, S_{t_2}, \ldots, S_{t_{24}}$ using the discretized updating formula in Equation (8), where $\Delta t = 1/12$. We simulate the index's price path $J = 50,000$ times and use them to calculate $R_T^j, j = 1, 2, \ldots, 50,000$. We input each R_T^j into the mapping function (Equation (1)) to obtain the structured product's return $f(R_T^j)$. The Buffered PLUS' fair value is the present value of the average $f(R_T^j)$. Depending on the random number seed we use, the value of the product ranges from \$87.30 to \$87.70.

Using the Numerical Integration Approach

To apply the numerical integration approach, we first define the function we want to integrate. In this case, the function is

[17] Summary information for this product is available at
`http://www.sec.gov/Archives/edgar/data/895421/000095010308003056/dp12147_424b2-ps11.htm`

$$\int_{-1}^{\infty} f(R_T) pdf(R_T) dR_T, \tag{14}$$

where

$$f(R_T) = \min(R_T + Buffer, 0) + \min(\alpha R_T, Cap) - \min(\alpha R_T, 0)$$

and $pdf(R_T)$ is the distribution from Equation (6).

We use a numerical integration method with adaptive Lobatto quadrature provided in Matlab. We require a tolerance level of 10e-10 and estimate the fair value of the product to be \$87.52.

Using the Decomposition Approach

As described in Section , we apply the decomposition approach by calculating the fair value of each of the product's components. The Buffered PLUS' payoff rule, described previously in Equation (1), can be rewritten in terms of equity derivatives and a zero-coupon bond as follows:

$$\begin{aligned}
P(S_T) &= I(1 + f(R_T)) \\
&= I(1 + \min(R_T + Buffer, 0) + \min(\alpha R_T, Cap) - \min(\alpha R_T, 0)) \\
&= I\left(1 + \min\left(\frac{S_T - S_0}{S_0} + Buffer, 0\right) + \min\left(\alpha \frac{S_T - S_0}{S_0}, Cap\right) - \min\left(\alpha \frac{S_T - S_0}{S_0}, 0\right)\right) \\
&= \underbrace{I(1 + Cap)} + \underbrace{\frac{I}{S_0} \min(S_T - (S_0 - S_0 Buffer), 0)} \\
&\quad + \underbrace{\frac{\alpha I}{S_0} \min\left(S_T - \left(S_0 + \frac{S_0 Cap}{\alpha}\right), 0\right)} - \underbrace{\frac{\alpha I}{S_0} \min(S_T - S_0, 0)}
\end{aligned} \tag{15}$$

By recalling that the payoff rule of a put option has the form $P(S_T) = \max(K - S_T, 0)$, we can decompose the Buffered PLUS payoff into four components:

1. A zero-coupon bond with a face value of $I(1 + Cap)$.
2. $\frac{I}{S_0}$ short put options with a strike price K of $S_0 - S_0 Buffer$.
3. $\frac{\alpha I}{S_0}$ short put options with a strike price of $S_0 + \frac{S_0 Cap}{\alpha}$.
4. $\frac{\alpha I}{S_0}$ long put options with a strike price of S_0.

For this Buffered PLUS, the zero-coupon bond has a face value of \$160 ($I(1 + Cap) = 100(1 + 60\%) = \$160$). The fair value of this component is

$$FairValue = e^{(-CDS + r)T} I(1 + Cap)$$

$$= e^{-(5.209\%+0.8496\%)2} \cdot \$160$$
$$= \$141.74. \quad (16)$$

The second component is 0.1159 shares ($\frac{I}{S_0} = \frac{100}{863.16} = 0.1159$) of short put options with a strike price of 776.84 ($S_0 - S_0 Buffer = 863.16 - 863.16 \cdot 10\% = 776.84$). We then use the Black-Scholes formula to calculate the option's fair value. The fair value of one put option with these parameters is \$131.71, so the fair value of 0.1159 put options is \$15.26.

The third component is 0.2317 shares ($2\frac{I}{S_0} = 2 \cdot 0.1159 = 0.2317$) of short put options with a strike price of 1122.11 ($K = S_0 + \frac{S_0 Cap}{\alpha} = 863.16 + \frac{863.16 \cdot 60\%}{2} = 1122.11$). Using the Black-Scholes formula, we calculate the fair value of one put option with these parameters to be \$346.35, meaning the fair value of this component is \$80.25.

The final component is 0.2317 shares ($\frac{\alpha I}{S_0}$) of long put options at strike price of \$863.16. The fair value of one such option is \$178.20, indicating the fair value of this component is \$41.29.

We combine the fair values of the four components to calculate the fair value of the Buffered PLUS.

$$\$141.74 - \$15.26 - \$80.25 + \$41.29 = \$87.52$$

Presenting the Buffered PLUS' payoff as a combination of options and a simple debt instrument can help investors understand that the Buffered PLUS is heavily exposed to the downside risk of the underlying security and the default risk of the issuer. The decomposition also shows the investor that a portfolio of a zero-coupon bond and put options on the S&P 500 Index would mimic the payoffs and stock market exposures of the product, but with less counterparty risk and more liquidity. The same decomposition is represented graphically in Figure 3, included in the Appendix.

In order to calculate the delta, we need to assume that the investment is $I = S_0$, which means comparing the structured product investment to the investment in the underlying securities. The delta for a first component is 0, since the first component is a zero-coupon bond. Other components include 1 share of short in-the-money put option, 2 shares of short at-the-money put options and 2 shares of long out-the-money put options. The deltas for the four components are 0, -0.30, -1.05, and -0.73, respectively. The delta for the structured product is $0-(-0.30)-(-1.05)+(-0.73) = 0.63$.

Using the PDE Approach

The Black-Scholes equation is the core equation used to value structured products, with boundary conditions

$$V(S,T) = P(S_T), \quad V(0,t) = I Buffer\, e^{-(r+CDS)*(T-t)}, \quad V(S,t) \sim I(1+Cap) \text{ as } S \to \infty.$$

We adopt the 'dimensionless' approaches in Wilmott et al. (1994) to simplify the equation and boundary conditions. Transforming the variable set (S, t) to a new variable set (x, τ) by the following equations:

$$S = S_0 e^x, \quad t = T - \frac{2\tau}{\sigma^2}, \quad V(S,t) = S_0 e^{\alpha x + \beta\tau} u(x,\tau) + I\mathit{Buffer}\, e^{-(r+CDS)*(T-t)},$$

where the constants

$$k_1 = \frac{2(r-d)}{\sigma^2}, \quad \alpha = -\frac{1}{2}(k_1 - 1), \quad \beta = -\frac{1}{4}(k_1-1)^2 - \frac{2(r+CDS)}{\sigma^2}.$$

The Black-Scholes equation after the change of variables becomes

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2}, \qquad \text{for } -\infty < x < \infty, \quad \tau > 0.$$

The initial condition becomes

$$\begin{aligned} u(x,0) &= u_0(x) && (17)\\ &= e^{\frac{1}{2}(k_1-1)x}\frac{I(1+Cap-\mathit{Buffer})}{S_0} \\ &\quad + \frac{I}{S_0}\min\left(e^{\frac{1}{2}(k_1+1)x} - (1-\mathit{Buffer})e^{\frac{1}{2}(k_1-1)x}, 0\right) \\ &\quad + \frac{\alpha I}{S_0}\min\left(e^{\frac{1}{2}(k_1+1)x} - \left(1 + \frac{Cap}{\alpha}\right)e^{\frac{1}{2}(k_1-1)x}, 0\right) \\ &\quad - \frac{\alpha I}{S_0}\min\left(e^{\frac{1}{2}(k_1+1)x} - e^{\frac{1}{2}(k_1-1)x}, 0\right). && (18) \end{aligned}$$

This is a standard heat equation with solution:

$$u(x,\tau) = \frac{1}{2\sqrt{\pi\tau}}\int_{-\infty}^{\infty} u_0(s) e^{-(x-s)^2/4\tau} ds.$$

The product value at issuance, plugging back $\tau = \frac{1}{2}T\sigma^2$, is

$$V(S_0, 0) = S_0 e^{\frac{1}{2}T\sigma^2\beta} u(0, \frac{1}{2}T\sigma^2) + I\mathit{Buffer}\, e^{-(r+CDS)T}.$$

Calculating the integral inside $u(0, \frac{1}{2}T\sigma^2)$, the value matches that derived using the decomposition approach.

Notice that the fair value is exactly the same using the numerical integration approach, the decomposition approach, or the PDE approach. The fair value calculated using the simulation approach is different because it varies depending on the number of simulated price paths, J, and the random seed. As $J \to \infty$, the range of fair values provided by the simulation approach will converge to \$87.52.

Valuation Without Default Risk

In the example above, all four approaches yield a price of about \$88. If we recalculate the price assuming the probability of default is zero we get a price closer to \$97. This highlights the potentially important component that default risk has in the proper valuation of these investments. Given the level of complexity of the valuation and the non-linear aspect of default it is possible that part of the reason investors seem willing to purchase these investments at a substantial premium may be that they do not fully comprehend the cost associated with default risk.

Another way to highlight this possibility is by comparing identical or almost identical products issued by two different issuers that have substantially different default risk. For example, on December 31, 2007 both UBS and Morgan Stanley issued an essentially identical structured product, an Absolute Return Barrier Note (ARBN).[18]

ARBNs are structured products that guarantee to return principal to the investor and a participation if the reference index stays in between two barriers until maturity, as long as the issuer does not default on the note.[19] The two above notes have very similar parameters: they were issued on December 31, 2007 and mature on June 30, 2009. The underlying security is the S&P 500 index. They differ in two ways: First, the UBS note has a maximum return of 25% and the Morgan Stanley Note has a maximum return of 21%. Second, and most importantly, UBS had a CDS spread of only 0.216% while Morgan Stanley had a CDS spread of 1.27%. Given those two differences, one might expect the two notes to have a different price. However, they both were priced at an identical \$10 per note. Using our pricing algorithms we calculate that the value of the UBS note is \$9.65 per note and the Morgan Stanley one \$9.37 per note.

This example, and many other similar examples of identical notes issued by different underwriter with significant different default risks at the price, highlight the possibility that of the many parameters that set the value of these complex products, default risk may not be properly priced by investors, maybe partially explaining the popularity of these products that tend to sell at a premium on average.

[18] For the UBS prospectus see: `http://www.sec.gov/Archives/edgar/data/1114446/000139340107000374/v098098_69075-424b2.htm`, and for the Morgan Stanley prospectus see: `http://www.sec.gov/Archives/edgar/data/895421/000095010307003121/dp08060_424b2-ps443.htm`.

[19] See for example Deng et al. (2011a) for a discussion of these notes and their pricing.

Conclusions

Until recently it has been difficult to estimate the fair value of many types of structured products due to the complex nature of payoff rules. In this paper we have reviewed and demonstrated four approaches that can be used to estimate the fair value of a wide variety of structured products, highlighting the benefits and limitations of each approach. This work is based on and complements our daily valuation of over 20,000 US structured products described in Deng et al. (2014b), as well as our product-specific valuation experience in Deng et al. (2012), Deng et al. (2011b), Deng et al. (2011a), Deng et al. (2010), and Deng et al. (2009).

Like many other researchers, we have documented significant issue date mispricing of many types of structured products.[20] We have also demonstrated that the issue date mispricing is largely responsible for the poor ex–post performance of structured products in the aggregate. While this mispricing could be due to transaction costs, underwriting fees, or differences in cost of capital between issuers, it is likely that issuers price their products to secure a gross margin sufficient to cover these costs and leave a net profit. The approaches described here, complemented by the SEC's new fair value disclosures for structured products, could detail the components of issue date mispricing.

Currently, the numerical integration and decomposition approaches are able to value only a subset of structured products, whereas the PDE and simulation approaches can value many more kinds of products. We will be better able to use the numerical integration approach as we derive the probability density functions of increasingly complex payoff rules. Similarly, as closed form valuations become available for more derivative instruments, the number of structured product types that can be valued using the decomposition approach will increase. In the meantime, simulation approaches are becoming ever more sophisticated and can take advantage of improvements in computational efficiency, especially in parallel environments. While valuation must keep up with the enormous growth and innovation in the structured product market, the approaches outlined here offer a powerful set of tools to investors and industry practitioners.

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[20] See especially (Henderson and Pearson, 2010).

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Appendix

Figure 3: Decomposition of a Buffered PLUS structured product

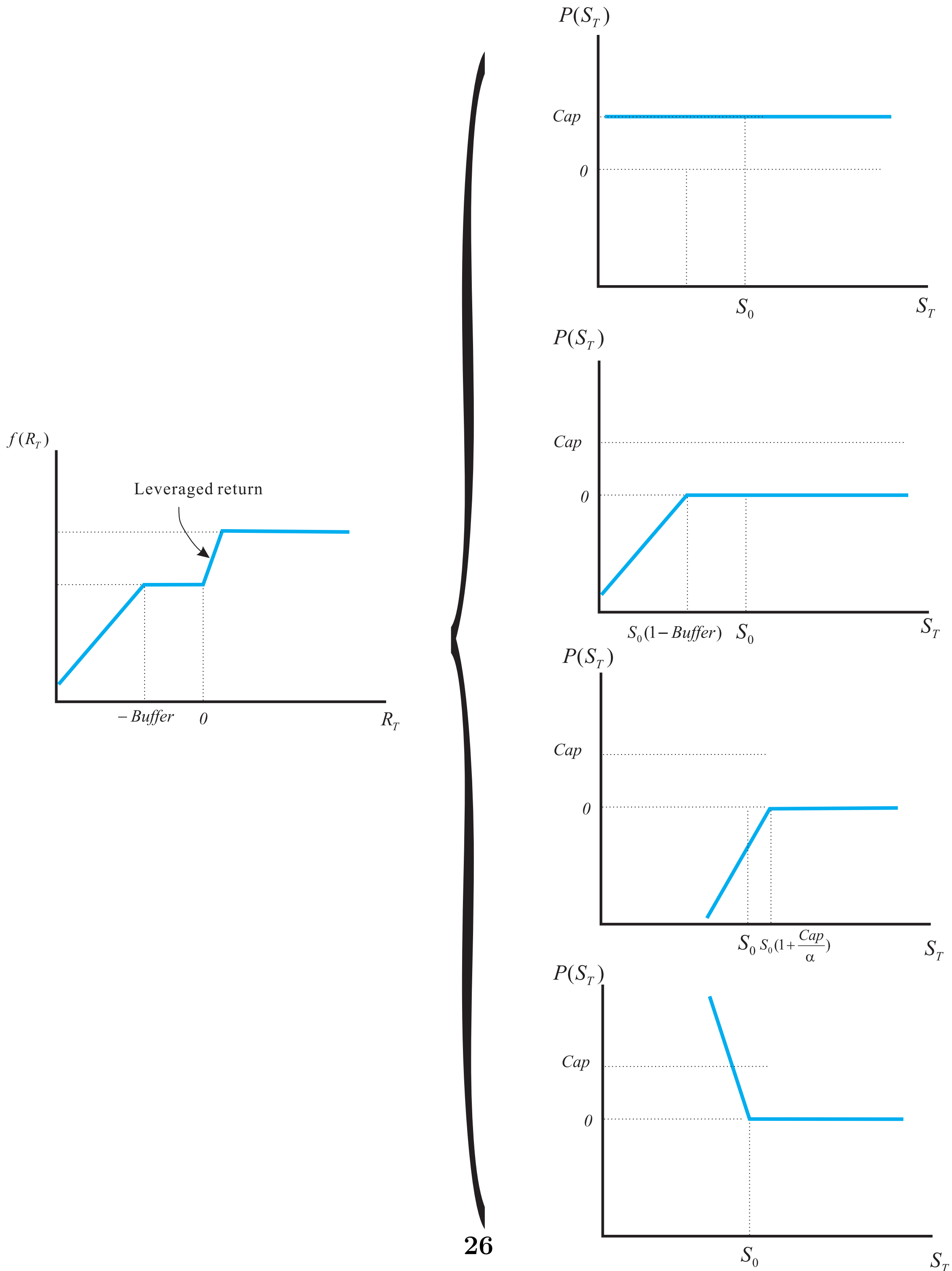
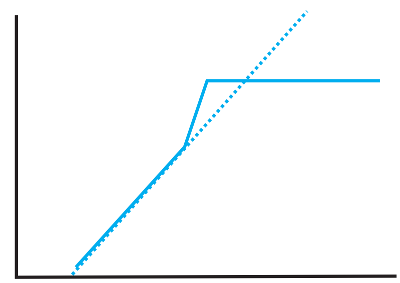
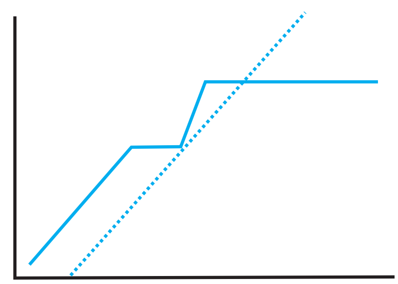
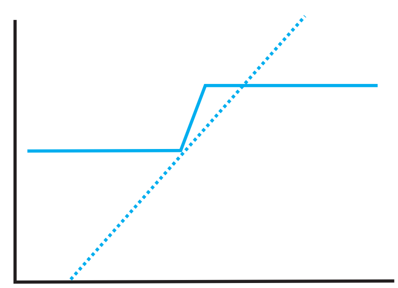


Table 2: Structured products that are linked to the ending levels of the underlying security

Structured Product	Mapping graph
PLUS (Performance Leverage Upside Security), Return Optimization Note, Stock Market Upturn Notes: The upside return is leveraged and capped, and the downside return is one to one to the underlying security. $f(R_T) = \begin{cases} \min(\alpha R_T, Cap), & \text{if } R_T \geq 0; \\ R_T, & \text{if } R_T < 0. \end{cases}$	
Buffered PLUS, Return Optimization with Partial Protection: The upside return is leveraged and capped. The downside return is partially protected by a return buffer. $f(R_T) = \begin{cases} \min(\alpha R_T, Cap), & \text{if } R_T \geq 0; \\ \min(R_T + \mathit{Buffer}, 0), & \text{if } R_T < 0. \end{cases}$	
Principal Protected Notes Principal is guaranteed to be paid back at maturity, which means the product has returns greater or equal to zero. The upside return is capped and occasionally leveraged. $f(R_T) = \begin{cases} \min(\alpha R_T, Cap), & \text{if } R_T \geq 0; \\ 0, & \text{if } R_T < 0. \end{cases}$	
Principal Protected Notes An un-capped version of the principal protection note. $f(R_T) = \begin{cases} \alpha R_T, & \text{if } R_T \geq 0; \\ 0, & \text{if } R_T < 0. \end{cases}$